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## Introduction

Hydrodynamic models of the North Sea are primarily concerned with tides and storm surges and the associated currents which can have velocities as high as several meters per second.

However the period of the dominant tide is only about a half day and the characteristic life time of a synoptic weather pattern is of the order of a few days. The very strong currents which are produced by the tides and the atmospheric forcing are thus relatively transitory and a Marine Biologist will argue that over time scales of biological interest, they change and reverse so many times that they more or less cancel out, leaving only a small residual contribution to the net water circulation.

The importance of tidal and wind induced currents on the generation of turbulence and the mixing of water properties is of course not denied but many biologists would be content with some rough parameterization of the efficiency of turbulent mixing and, for the rest, some general description of the long term transport of "water masses".

Although the concept of "moving water masses", and its train of pseudo-lagrangian misdoings, appeal to chemists and biologists who would like to find, in the field, near-laboratory conditions, it is impossible to define it in any scientific way and charts of the North Sea's waters like the one shown in figure 1 and reproduced from Laevastu (1963) are easily misinterpreted and often confuse the situation by superposing a flow pattern on an apparently permanent "geography" of water masses.

The notion of "residual" circulation - which, at least, has an Eulerian foundation - has long remained almost as vague. Some people have defined it as the observed flow minus the computed tidal flow. Such a definition is understandable from a physical point of view but one must realize that the residual flow so-defined contains all wind-induced currents, including small scale fluctuations. It is definitely not a steady or quasi-steady flow and some attempts to visualize it by means of streamlines are questionable.
What it represents, in terms of marine chemistry or marine ecology is not at all clear.

Actually, if one wants to take the point of view of the marine ecologist, what one should really look at is the *mean flow* over some appropriate period of time of biological interest.

It is customary for experimentalists to compute, from long series of observations, daily, weekly and monthly averages.

What such averages actually represent is debatable.

No doubt that tidal currents are essentially removed in this process. However with tidal velocities, one or two orders of magnitude higher than residual velocities and the latter of the order of traditional current-meters'errors, one may fear that, as a result of the non-linearities of the equipment, the error remains the same order of magnitude after averaging and leads to a 100 % inaccuracy in the calculated mean residual (*e.g.* Nihoul, 1980).

Moreover the choice of the periods of time over which the averages are made is not obvious as it seems to rely more on the calendar than on physical processes. One must be quite clear of what one gets from such averages.

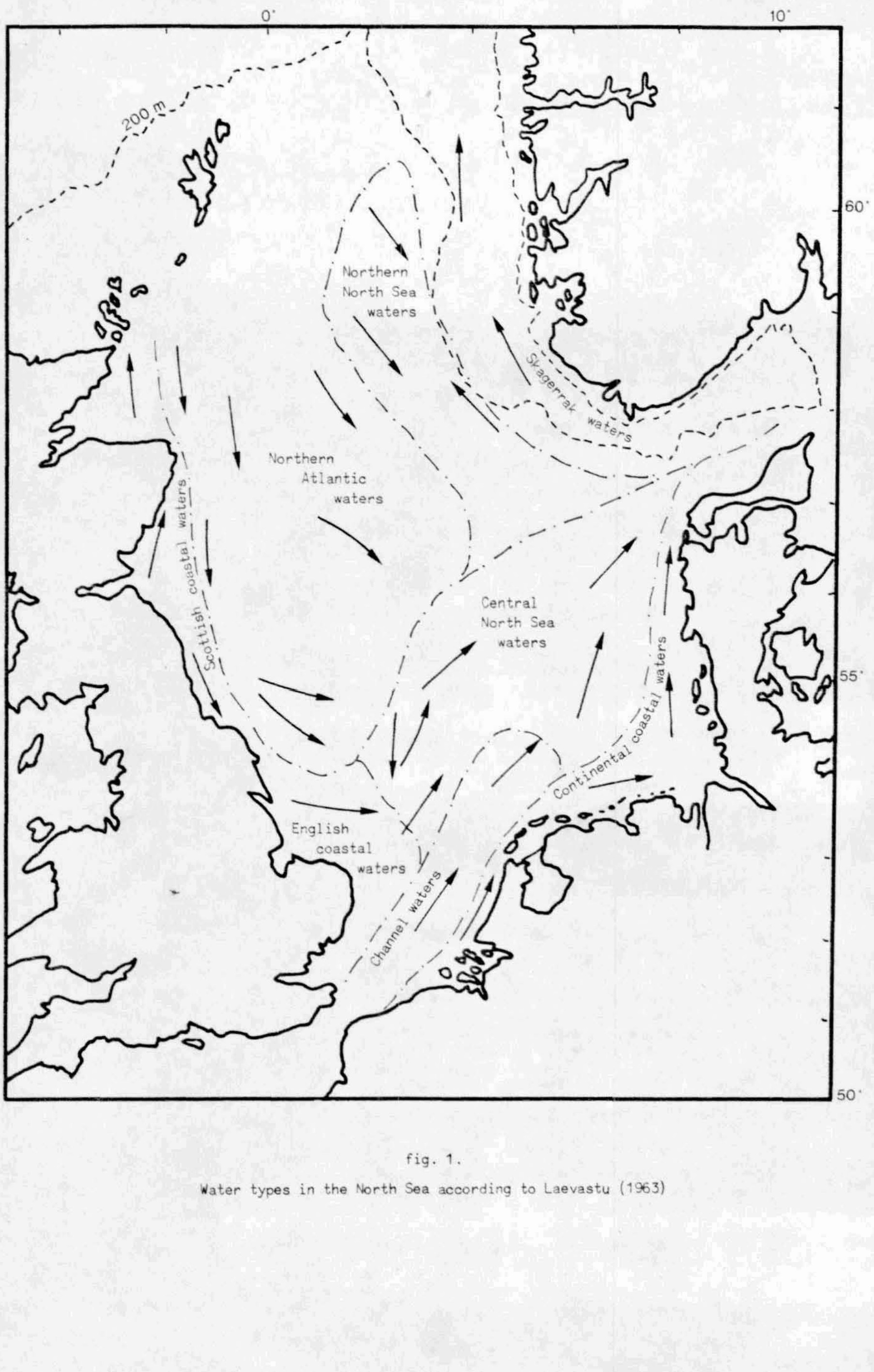


fig. 1.
Water types in the North Sea according to Laevastu (1963)

With tides reversing four times daily and changes in the synoptic weather pattern taking several days, one may expect daily averages to remove tidal motions while still catching most of the residual currents responding to the evolving meteorological conditions.

Monthly averages, on the other hand, will have a more "climatic" sense and will presumably represent the residual circulation which is induced by macroscale oceanic currents and the mean effect of non-linear interactions of mesoscale motions (tides, storm surges ...).

The role of residual currents and residual structures (fronts ...) in the dynamics of marine populations, the long term transport of sediments or the ultimate disposal of pollutants, for example, is universally recognized but different schools of theoreticians and experimentalists still favour different definitions which, in the case of the North Sea, may have little in common apart from the fact that the strong tidal oscillations have been removed.

Obviously, each definition addresses a particular kind of problem and if, as it is now universally agreed, the residual circulation is defined as the mean motion over a period of time sufficiently large to cancel tidal oscillations and transient wind-induced currents, there is still the problem of choosing the time interval of averaging, taking into account the objectives of the study.

In any case, it is not demonstrated that such a time average may be obtained with sufficient accuracy from experimental records. As pointed out before, the averaging takes away more than 90 % of the signal and the final result is of the same order as the instrumental error.

In the following, one examines how the problem can be approached through mathematical modelling.

## The governing equations

The three-dimensional hydrodynamic equations applicable to a well-mixed continental sea, like the North Sea, can be written (*e.g.* Nihoul, 1975)

$$\nabla . \mathbf{v} = 0 \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla . (\mathbf{v}\mathbf{v}) + 2\, \boldsymbol{\Omega} \wedge \mathbf{v} = - \nabla q + \nabla . \mathbf{R} \tag{2}$$

where $\boldsymbol{\Omega}$ is the Earth's rotation vector , $q = \frac{P}{\rho} + g x_3$ , P is the pressure, $\rho$ the specific mass of sea water, $x_3$ the vertical coordinate and R the Reynolds stress tensor (the stress is here per unit mass of sea water) resulting from the non-linear interactions of three-dimensional microscale turbulent fluctuations.

The Reynolds stress tensor can be parameterized in terms of eddy viscosity coefficients. In microscale three-dimensional turbulence, these coefficients are of the same order of magnitude in the horizontal and vertical directions. Then, horizontal length scales being much larger than the depth, the last term in the right-hand side of eq.(2) can be written simply, with a very good approximation

$$\nabla . \mathbf{R} = \frac{\partial \boldsymbol{\tau}}{\partial x_3} = \frac{\partial}{\partial x_3} \left( \tilde{\nu} \frac{\partial \mathbf{v}}{\partial x_3} \right) \tag{3}$$

where $\tilde{\nu}$ is the vertical eddy viscosity and $\boldsymbol{\tau}$ the Reynolds stress (vector).

The residual flow is defined as the mean flow over a time T sufficiently large to cover at least one or two tidal periods. If the subscript "$_0$" denotes such an average, one may write

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1 \tag{4}$$

with

$$\frac{1}{T} \int_t^{t+T} \mathbf{v} \, dt = \mathbf{v}_0 \tag{5}$$

$$\frac{1}{T} \int_t^{t+T} \mathbf{v}_1 \, dt = 0 \tag{6}$$

What $v_0$ and $v_t$ respectively include depends on the time of integration T.

If T is of the order of one day (exactly two or three periods of the dominant $M_2$ tide), $T \sim 10^5$, the averaging eliminates the tidal currents and smoothes out all current fluctuations, -generated by variations of the wind field, for instance- , which have a time smaller than T.

However, as mentioned before, changes in the synoptic weather pattern have time scales comparable with $T \sim 10^5$. Thus, unless one considers periods of negligible meteorological forcing, $T \sim 10^5$ does not correspond to a valley in the energy spectrum of the currents. In that case, one cannot derive an equation for $v_0$ by averaging eq.(2) and assuming that, as for an ensemble average, the averaging commutes with the time derivative. Furthermore, $v_0$ defined in this way, depends very much on time and doesn't correspond to the quasi-steady drift flow the biologists have in mind when they talk about residuals.

One might argue that such a time dependent daily mean is still worth calculating to follow the response of the sea to the evolving weather pattern especially in storm conditions. However a time step of the order of $10^5$ is too large to predict the storm-induced currents with accuracy and it is much wiser, in that case, to forget about averaging and solve eq.(1) and (2) for tides and storm surges simultaneously.

Thus, "daily" residuals do not seem to be appropriate to describe real situations in the North Sea.

From a mathematical point of view, however, one can always consider the mean currents over two or three tidal periods, neglecting all atmospheric influence. Such "tidal residuals" emphasize the part played by tidal motions in determining the residual circulation and, with very much less computer work needed, they give a fairly good idea of the "climatic residual circulation" described below.

If one takes, now, a much greater time of averaging, say T of the order of $10^6$ ($\sim$ two weeks) to $10^7$ ($\sim$ four months) one may expect, over such a long time, a great variety of different meteorological conditions resulting in an almost random atmospheric forcing on the sea. The current patterns will reflect the atmospheric variability and, on the average, there will be only a small residue.

The mean flow over a time $T \sim 10^6, 10^7$ may be regarded as the "climatic residual" flow which affects the dynamics of biological populations, the long term transport of sediments and the slow removal of pollutants.

The climatic circulation in the North Sea is produced by the inflow and outflow of macroscale Atlantic currents, by the action of the mean wind stress and, as shown below, by the mean effect of non-linear interactions of mesoscale motions (tides, storm surges ...).

The equations for the climatic residual flow may be obtained by taking the average of eqs.(1) and (2) over the chosen time $T$ ($T \sim 10^6, 10^7$).

The time derivative in the left-hand side of eq.(2) gives a contribution

$$\frac{\mathbf{v}(t+T) - \mathbf{v}(t)}{T} \tag{7}$$

Since the time T has been chosen a multiple of the main tidal period and large enough to cover a great variety of meteorological events, one should expect the numerator of (7) to be of the same order as the residual velocity $\mathbf{v}_0$. If one takes it to be one order of magnitude larger to be on the safe side, one find

$$\frac{\mathbf{v}(t+T) - \mathbf{v}(t)}{T} \lesssim 0(10^{-5}\, v_0) \tag{8}$$

The average of the Coriolis acceleration is

$$2\,\mathbf{\Omega} \wedge \mathbf{v}_0 \sim 0(10^{-4}\, v_0)$$

One may thus neglect the contribution of the time derivative in the equation for $\mathbf{v}_0$ .

The climatic residual circulation is then given by the steady state equations

$$\nabla . \mathbf{v}_0 = 0 \tag{10}$$

$$\nabla . (\cancel{\mathbf{v}_0 \mathbf{v}_0}) + 2\,\Omega \wedge \mathbf{v}_0 = - \nabla q_0 + \frac{\partial \tau_0}{\partial x_3} + \nabla . \mathbf{N} \tag{11}$$

where

$$\mathbf{N} = (- \mathbf{v}_1 \mathbf{v}_1)_0 \tag{12}$$

Since $\mathbf{v}_0$ is one or two orders of magnitude smaller than $\mathbf{v}_1$ which contains in particular the tidal currents, the first term in the left-hand side of eq.(12) is completely negligible. The tensor **N** in the right-hand side plays, for mesoscale motions, a role similar to that of the turbulent Reynolds stress tensor **R** in eq.(2) and may be called the "mesoscale Reynolds stress tensor". The last term in the right-hand side of eq.(11) represents an additional force acting on the residual flow and resulting from the non-linear interactions of mesoscale motions (tides, storm surges ...).

The importance of this force was discovered, first, by depth-integrated numerical models of the residual circulation in the North Sea (Nihoul 1974, Nihoul and Ronday 1975) and the associated stress was initially referred to as the "tidal stress" to emphasize the omnipresent contribution of tidal motions.

## The mesoscale Reynolds stress tensor

The tensor **N** can be computed explicitly by solving eqs. (1) and (2) for mesoscale motions and taking the climatic average of the dyadic $\mathbf{v}_1 \mathbf{v}_1$ .

In fact the solution of eqs. (1) and (2) with appropriate wind forcing and open sea boundary conditions yields

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1$$

and one may reasonably ask the question why one must go through the processe of computing $\mathbf{N}$ and solving eqs. (10) and (11) to obtain the residual velocity $\mathbf{v}_0$ *i.e.* why one cannot solve (1) and (2) for the total velocity $\mathbf{v}$ and simply derive $\mathbf{v}_0$ from $\mathbf{v}$ *directly*, by averaging the solution of eqs. (1) and (2).

The problem here, again, is that, in the North Sea, $\mathbf{v}_1$ represents 90 % of $\mathbf{v}$. If one allows for an error $\delta\mathbf{v}$ on $\mathbf{v}$ of, say, 10 %, resulting from the imprecision of open sea boundary conditions and from the approximations of the numerical method, the error is of the same order of magnitude as the residual flow $\mathbf{v}_0$ .

Because of non-linearities, one may fear that, in the averaging process, this error does not, for the essential, cancel out as $\mathbf{v}_1$ does. Thus averaging the solution $\mathbf{v}$ of eqs. (1) and (2), one gets $\mathbf{v}_0 + (\delta\mathbf{v})_0$ *i.e.* the residual velocity with an error which may be as large as 100 % (Nihoul and Ronday, 1976a).

The procedure is conceivable when modelling a very limited area (near a coast, for instance) where the mesh size of the numerical grid can be reduced and where the open-sea boundary conditions can be determined with greater accuracy by direct measurements. Then $\delta\mathbf{v}$ can be made small enough for the average $\mathbf{v}_0 + (\delta\mathbf{v})_0$ to provide a satisfactory evaluation of the residual flow $\mathbf{v}_0$ .

In the case of the North Sea or, even, the Southern Bight or the English Channel, models of such a high accuracy are prohibitively expensive and cannot be considered for routine forecasting.

However, the classical models give $\mathbf{v}_1$ with a fair accuracy and they can be used to compute the mesoscale stress tensor $\mathbf{N}$ .

The latter can be substituted in eq. (11) and the system of eqs. (10) and (11) can be solved very quickly to obtain $\mathbf{v}_0$ .

One can show that, in this way, one can determine $\mathbf{v}_0$ with good accuracy.

Typical values for the North Sea show that, in general, the two terms $2\,\mathbf{\Omega} \wedge \mathbf{v}_0$ and $\nabla.(-\,\mathbf{v}_1\mathbf{v}_1)_0$ are of the same order of magnitude.

If $\delta \mathbf{v}_1$ is the error on $\mathbf{v}_1$, one has

$$\delta[\nabla.(-\mathbf{v}_1\mathbf{v}_1)_0] \sim [\nabla.(-\mathbf{v}_1\mathbf{v}_1)_0]\frac{\delta v_1}{v_1}$$

$$\sim 0(2\Omega v_0 \frac{\delta v_1}{v_1})$$

This error induces an error $\delta \mathbf{v}_0$ on $\mathbf{v}_0$ given by

$$2\,\mathbf{\Omega} \wedge \delta \mathbf{v}_0 \sim \delta[\nabla.(-\mathbf{v}_1\mathbf{v}_1)_0] \sim 0(2\Omega v_0 \frac{\delta v_1}{v_1})$$

*i.e.*

$$\frac{\delta v_0}{v_0} \sim \frac{\delta v_1}{v_1}$$

Hence the *relative* error is the same on $\mathbf{v}_0$ and on $\mathbf{v}_1$ and not the *absolute* error as before. Thus if $v_1$ can be computed with, say, a 90 % precision, the solution of the averaged equations (10) and (11) will give the residual circulation with the same 90 % precision.

## The equation for the horizontal transport

If one writes

$$\mathbf{v} = \mathbf{u} + w\,\mathbf{e}_3 \quad ; \quad \mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1 \qquad (13);(13')$$

emphasizing the horizontal velocity vector $\mathbf{u}$, one defines the residual horizontal transport as

$$\mathbf{U}_0 = \int_{-h}^{\zeta_0} \mathbf{u}_0 \, dx_3 = H_0 \, \overline{\mathbf{u}}_0 \qquad (14)$$

where $\overline{\mathbf{u}}_0$ is the depth-averaged velocity, $H_0 = h + \zeta_0$, $h$ is the depth and $\zeta_0$ the residual surface elevation. ($H_0 \sim h$ because $\zeta_0 \ll h$).

The derivation of equations for the residual transport by integration of eqs. (10) and (11) over depth is quite straightforward (*e.g.* Nihoul, 1975a). One finds, after some reordering,

$$\nabla . \mathbf{U}_0 = 0 \tag{15}$$

$$f \, \mathbf{e}_3 \wedge \mathbf{U}_0 = - H_0 \nabla q_0 - K \, \mathbf{U}_0 + \boldsymbol{\Theta} \tag{16}$$

where

$$K = \frac{D \, \| \bar{\mathbf{u}}_1 \|_0}{H_0} \tag{17}$$

$\bar{\mathbf{u}}_1$ denoting the depth-mean of $\mathbf{u}_1$ and $\boldsymbol{\Theta}$ standing in brief for

$$\boldsymbol{\tau}_0^s + \boldsymbol{\tau}_0^n - \boldsymbol{\tau}_0^b$$

where

(i) $\boldsymbol{\tau}_0^s$ is the residual wind stress

(ii) $\boldsymbol{\tau}_0^n$ is the mesoscale Reynolds stress

$$\boldsymbol{\tau}_0^n = \int_{-h}^{\zeta_0} \nabla . (- \mathbf{v}_1 \mathbf{u}_1)_0 \, dx_3 \tag{18}$$

(iii) $\boldsymbol{\tau}_0^b$ is the mesoscale "friction stress"

$$\boldsymbol{\tau}_0^b = (D \, \| \bar{\mathbf{u}}_1 \| \, \bar{\mathbf{u}}_1)_0 \tag{19}$$

The friction stress is the part of the residual bottom stress (the first part is $- K \, \mathbf{U}_0$) which results from the non-linear interactions of mesoscale motions. It is analogous to the Reynolds stress $\boldsymbol{\tau}_0^n$ and represents an additional forcing on the residual flow.

Since $\mathbf{U}_0$ is a two-dimensional horizontal vector, eq. (15) suggests the introduction of a stream function $\psi(x_1, x_2)$ such that

$$U_{0.1} = - \frac{\partial \psi}{\partial x_2} \tag{20}$$

$$U_{0.2} = \frac{\partial \psi}{\partial x_1} \tag{21}$$

Eliminating $q_0$ between the two horizontal components of eq. (16), one obtains then a single elliptic equation for $\psi$, viz. [using (20)]

$$\frac{K}{h}\nabla^2\psi + \frac{\partial\psi}{\partial x_1}\left[\frac{\partial}{\partial x_1}\left(\frac{K}{h}\right) + \frac{\partial}{\partial x_2}\left(\frac{f}{h}\right)\right] + \frac{\partial\psi}{\partial x_2}\left[\frac{\partial}{\partial x_2}\left(\frac{K}{h}\right) - \frac{\partial}{\partial x_1}\left(\frac{f}{n}\right)\right] = \frac{\partial}{\partial x_1}\left(\frac{\theta_2}{h}\right) - \frac{\partial}{\partial x_2}\left(\frac{\theta_1}{h}\right) \tag{22}$$

This equation must be solved with appropriate boundary conditions. If one can simply take $\psi$ = const. along the coasts, the conditions on the open-sea boundaries are much more difficult to assess. One has estimates of the total inflows through the Straits of Dover [~ 7400 $km^3.y^{-1}$ (Van Veen, 1938; Carruthers, 1935)], the Northern boundary [~ 23000 $km^3.y^{-1}$ (Kalle, 1949; Laevastu, 1963)], through the Skagerrak [~ 479 $km^3.y^{-1}$ (Ices Skagerrak Expedition)] as well as of the contribution of the main rivers [~ 245 $km^3.y^{-1}$ (Mc Cave, 1974)] but the distribution of these flows along the boundaries are poorly known and one must resort to interpolation formulas which may or may not represent adequately the contribution, to the residual circulation of the North Sea, of inflowing or outflowing oceanic macroscale currents (*e.g.* Ronday, 1975). A better determination of the conditions along open-sea boundaries is needed and should be considered with the highest priority in the near future.

Eq.(22) shows the influence on the residual flow of the residual friction coefficient K and its gradient, of the distribution of depths, on the curl of the residual wind stress and of the mesoscale stresses $\tau_0^n$ and $\tau_0^l$.

In relatively coarse grid models of the whole North Sea (where the variations of K and h are partly smoothed out), the effect of the mesoscale stresses appears to be the most spectacular. This is illustrated by figures (2), (3) and (4), fifuring the residual circulation in negligible wind conditions. Figure (2) shows the residual flow pattern assuming a constant depth of 80 m and neglecting $\tau_0^n$ and $\tau_0^b$.

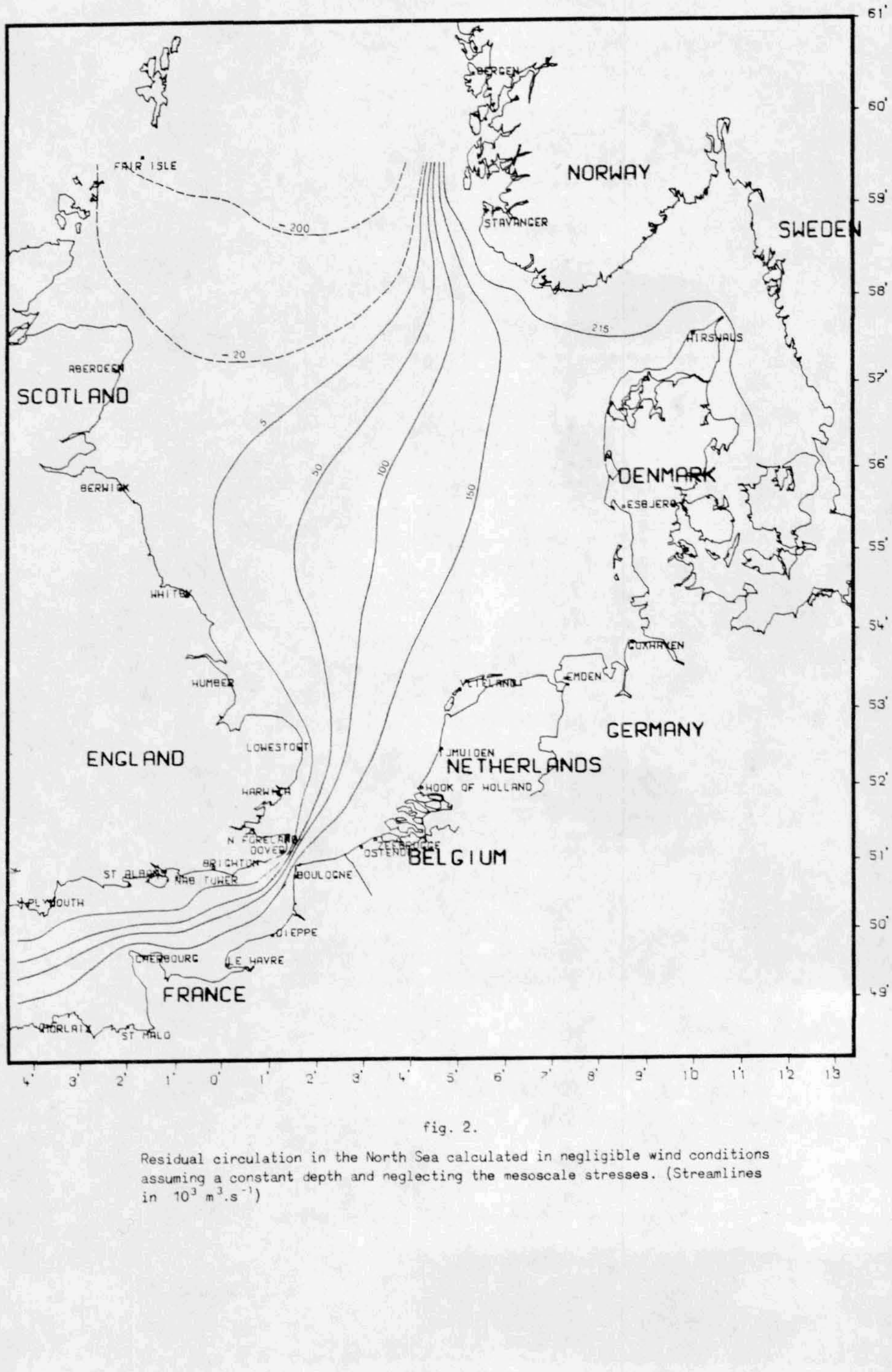


fig. 2.

Residual circulation in the North Sea calculated in negligible wind conditions assuming a constant depth and neglecting the mesoscale stresses. (Streamlines in $10^3\ m^3.s^{-1}$)

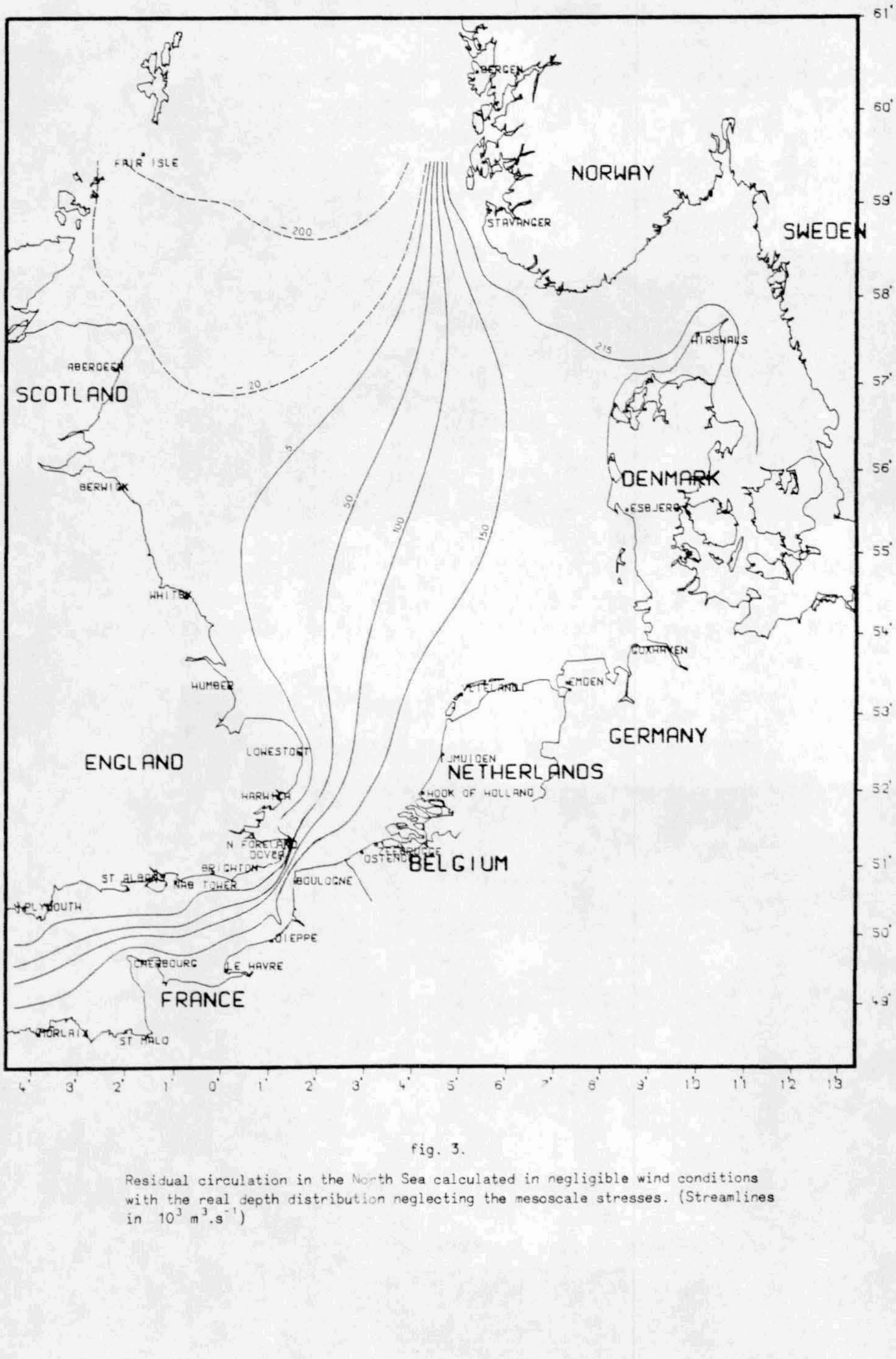


fig. 3.

Residual circulation in the North Sea calculated in negligible wind conditions with the real depth distribution neglecting the mesoscale stresses. (Streamlines in $10^3\ m^3.s^{-1}$)

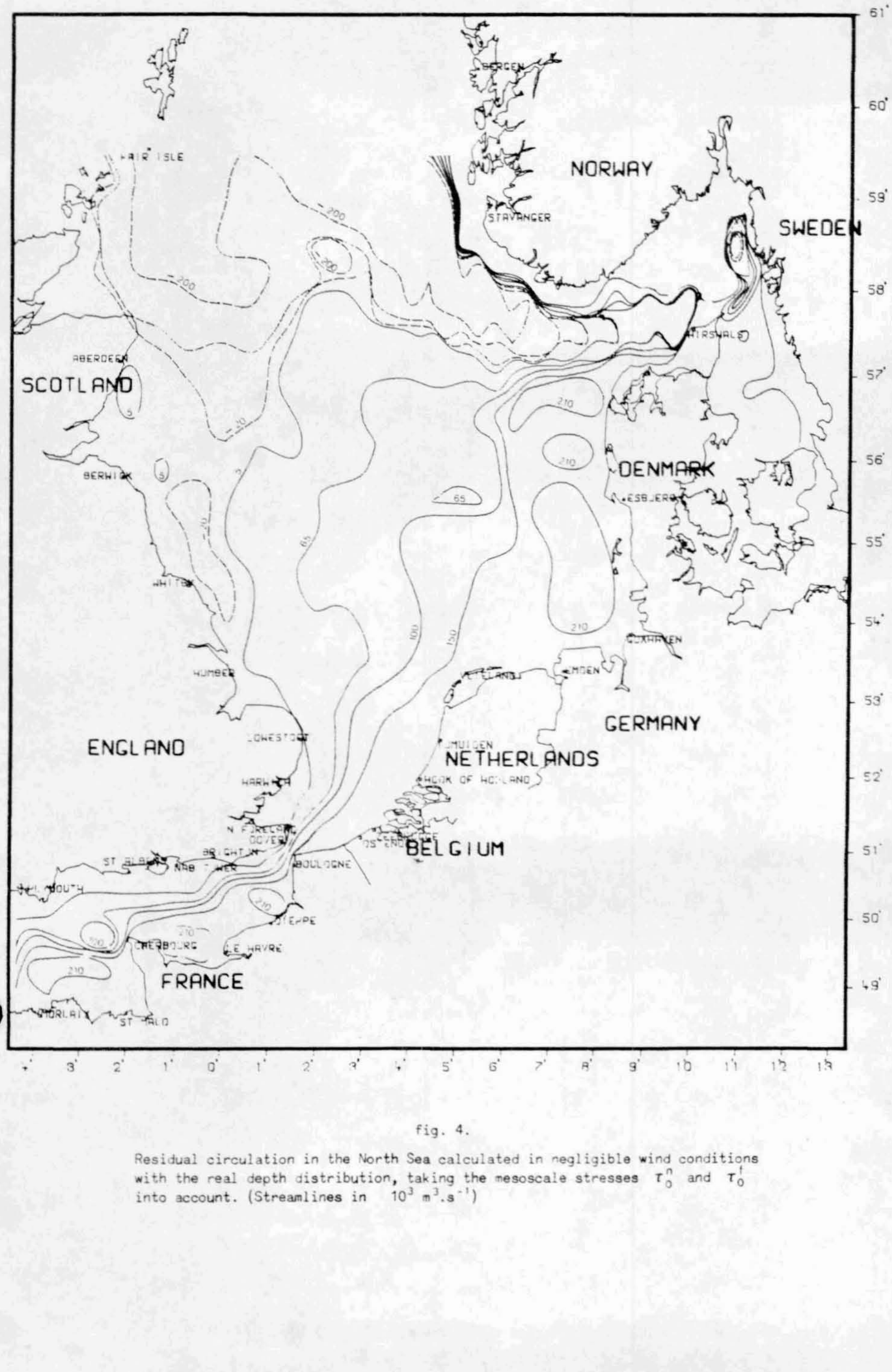


fig. 4.

Residual circulation in the North Sea calculated in negligible wind conditions with the real depth distribution, taking the mesoscale stresses $\tau_0^n$ and $\tau_0^t$ into account. (Streamlines in $10^3\ m^3.s^{-1}$)

Figure (3) shows the flow pattern taking the depth distribution into account and neglecting $\tau_0^n$ and $\tau_0^b$.
Figure (4) shows the flow pattern taking the depth distribution into account and including $\tau_0^n$ and $\tau_0^t$ computed from the results of a preliminary time dependent model of mesoscale flows.

The differences between figures (2) and (3) are small. They both reproduce the broad trend of the residual circulation induced by the in-and out-flow of two branches of the North Atlantic current but they fail to uncover residual gyres which constitue essential features of the residual flow pattern and which have been traced in the field by observations (*e.g.* Zimmerman, 1976; Riepma, 1977; Beckers *et al.*, 1976).

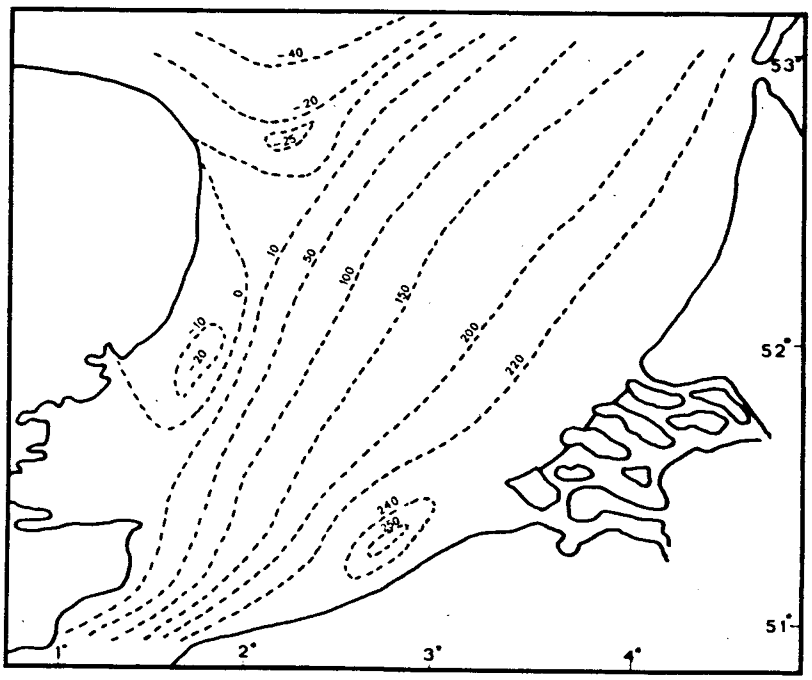


fig. 5.

Residual circulation in the Southern Bight calculated in negligible wind conditions, with the real depth distribution, taking into account the mesoscale stresses $\tau_0^n$ and $\tau_0^t$. (Streamlines in $10^3\ m^3.s^{-1}$)

A comparison between figure (4) and figure (1) shows a good agreement between the predictions of the model and the expected circulation of water masses in the North Sea.

However, as mentioned before, a model covering the whole North Sea does not have a sufficiently fine resolution (of bottom topography, for instance) and cannot detect all the existing gyres.

For that reason, three models were run simultaneously, one covering the North Sea and part of the English Channel, another one, the Southern Bight and the third one, the Belgian coastal waters; the large scale models providing open-sea boundary conditions for the smaller scale models.

Figure (5) shows the residual circulation in the Southern Bight. One notices in particular a gyre off the Belgian coast which was not apparent on figure (4). This gyre is produced by the mesoscale stresses in relation with the spatial variations of the depth and of the residual friction coefficient K (Nihoul and Ronday, 1976). The presence of the gyre has been shown to play an important role in the distribution of sediments (Nihoul, 1975b) and in creating off the Northern Belgian coast the conditions of an outer-lagoon with specific chemical and ecological characteristics (Nihoul, 1974; Beckers *et al.*, 1976).

## The energy equations

Using eq. (3), one can write the equations for $\mathbf{v}$ , $\mathbf{v}_0$ and $\mathbf{v}_1$ in the form

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla.(\mathbf{v}\mathbf{v}) + 2\,\Omega \wedge \mathbf{v} = -\nabla q + \frac{\partial \tau}{\partial x_3} \tag{23}$$

$$\nabla.(\mathbf{v}_0\mathbf{v}_0) + 2\,\Omega \wedge \mathbf{v}_0 = -\nabla q_0 + \frac{\partial \tau_0}{\partial x_3} + \nabla.\mathrm{N} \tag{24}$$

$$\frac{\partial \mathbf{v}_1}{\partial t} + \nabla.[\mathbf{v}_1\mathbf{v}_0 + \mathbf{v}_0\mathbf{v}_1 + \mathbf{v}_1\mathbf{v}_1 - (\mathbf{v}_1\mathbf{v}_1)_0] + 2\,\Omega \wedge \mathbf{v}_1 = -\nabla q_1 + \frac{\partial \tau_1}{\partial x_3} \tag{25}$$

with

$$\nabla . \mathbf{v} = \nabla . \mathbf{v}_0 = \nabla . \mathbf{v}_1 = 0 \qquad (26),(27),(28)$$

One can see that the equation for $\mathbf{v}_1$ is essentially the same as the equation for $\mathbf{v}$. They only differ by terms which are orders of magnitude smaller. It is the reason why, one can, with the appropriate boundary conditions, determine the mesoscale velocity $\mathbf{v}_1$, in a first step, and the residual velocity $\mathbf{v}_0$, in a second step, taking the coupling between the two types of motion into account in the calculation of $\mathbf{v}_0$ only.

Taking the scalar products of eqs. (23), (24) and (25) respectively by $\mathbf{v}$, $\mathbf{v}_0$ and $\mathbf{v}_1$, using (26), (27) and (28), and averaging over $T$, one finds, neglecting again the contributions from the time derivatives under the assumption that $T$ is sufficiently large :

$$\nabla . (\mathbf{v} \frac{\mathbf{v}^2}{2} + \mathbf{v}\, q)_0 = \frac{\partial}{\partial x_3} (\mathbf{v} . \tau)_0 - (\tau . \frac{\partial \mathbf{v}}{\partial x_3})_0 \qquad (29)$$

$$\nabla . (\mathbf{v}_0 \frac{\mathbf{v}_0^2}{2} + \mathbf{v}_0 q_0 - \mathbf{v}_0 . \mathbf{N}) = \frac{\partial}{\partial x_3} (\mathbf{v}_0 . \tau_0) - \tau_0 . \frac{\partial \mathbf{v}_0}{\partial x_3} - \mathbf{N} : \nabla \mathbf{v}_0 \qquad (30)$$

$$\nabla . (\mathbf{v}_0 \frac{\mathbf{v}_1^2}{2} + \mathbf{v}_1 \frac{\mathbf{v}_1^2}{2} + \mathbf{v}_1 q_1)_0 = \frac{\partial}{\partial x_3} (\mathbf{v}_1 . \tau_1)_0 - (\tau_1 . \frac{\partial \mathbf{v}_1}{\partial x_3})_0 + \mathbf{N} : \nabla \mathbf{v}_0 \qquad (31)$$

The terms in the left-hand sides of eqs. (29), (30) and (31) are of the divergence form. They represent fluxes of energy in physical space. The terms in the right-hand sides represent rates of energy production or destruction or energy exchanges between scales of motion.

Integrating over depth, one can see for instance that the first terms represent the average rate of work of the wind stress $\tau^s$, *i.e.*

$$(\mathbf{v}^s . \tau^s) = \mathbf{v}_0^s . \tau_0^s + (\mathbf{v}_1^s . \tau_1^s)_0 \qquad (32)$$

where $\mathbf{v}^s$ denotes the surface velocity.

The second term in the right-hand side of eq. (29) is related to the average dissipation of energy. Using eq. (3), one has, indeed

$$- \boldsymbol{\tau} \cdot \frac{\partial \mathbf{v}}{\partial x_3} = - \tilde{\nu} \left\| \frac{\partial \mathbf{v}}{\partial x_3} \right\|^2 \tag{33}$$

where $\tilde{\nu}$ is the eddy viscosity.

The depth-averaged dissipation rate

$$\varepsilon = \frac{1}{H_0} \int_{-h}^{\zeta_0} \left( \boldsymbol{\tau} \cdot \frac{\partial \mathbf{v}}{\partial x_3} \right)_0 dx_3 \tag{34}$$

can be split in two parts, as seen from eqs. (30) and (31), *i.e.*

$$\varepsilon = \frac{1}{H_0} \int_{-h}^{\zeta_0} \boldsymbol{\tau}_0 \cdot \frac{\partial \mathbf{v}_0}{\partial x_3} dx_3 + \frac{1}{H_0} \int_{-h}^{\zeta_0} \left( \boldsymbol{\tau}_1 \cdot \frac{\partial \mathbf{v}_1}{\partial x_3} \right)_0 dx_3 \tag{35}$$

## The contribution of the residual stress $\boldsymbol{\tau}_0$ to the energy budget

The second term in the right-hand side of eq.(35) is obviously related to the energy dissipated by the mesoscale motions. It is, by far, the essential contribution to $\varepsilon$ and may serve as a first approximation of it. It is however the first term one is interested in, here, to explain the physical mechanisms which contribute to shape the residual circulation.

In evaluating this term, one can obviously restrict attention to the horizontal components of the vectors $\boldsymbol{\tau}_0$ and $\mathbf{v}_0$ ,and this is also true for any scalar product of the form $\boldsymbol{\tau} \cdot \frac{\partial \mathbf{v}}{\partial x_3}$

One has indeed, from the continuity equation,

$$\frac{\partial w}{\partial x_3} \leq \nabla . \mathbf{u} \sim O\left(\frac{u}{L}\right)$$

where $L$ is the characteristic scale of horizontal variations. On the other hand

$$\frac{\partial \mathbf{u}}{\partial x_3} \sim O\left(\frac{u}{h}\right)$$

Since $h \ll L$, the contributions from the vertical velocity are completely negligible in the integrals of eq. (35).

The application of the three-dimensional equations (1) and (2) to the North Sea (Nihoul, 1977; Nihoul *et al.*, 1979) shows that

(i) the turbulent stress can be written

$$\boldsymbol{\tau} = \boldsymbol{\tau}^s \xi + \boldsymbol{\tau}^b (1 - \xi) + \kappa \, \|\boldsymbol{\tau}^b\|^{\frac{1}{2}} \, H \sum_1^{\infty} A_n \, \lambda(\xi) \, \frac{df_n}{d\xi} \tag{36}$$

where $\boldsymbol{\tau}^s$ and $\boldsymbol{\tau}^b$ are respectively the surface stress and the bottom stress (per unit mass of sea water), $\xi = H^{-1}(x_3 + h)$, $H = h + \zeta$, $h$ is the depth and $\zeta$ the surface elevation, the $A_n$'s are functions of $t$, $x_1$ and $x_2$ involving $\boldsymbol{\tau}^s$, $\boldsymbol{\tau}^b$ and their time derivaties, $\kappa$ is the Von Karman constant,

$$\lambda(\xi) = \frac{\tilde{\nu}}{\kappa \, \|\boldsymbol{\tau}^b\|^{\frac{1}{2}} \, H}$$

and the functions $f_n(\xi)$ are the eigenfunctions of the problem

$$\frac{d}{d\xi}\left(\lambda \frac{df_n}{d\xi}\right) = -\alpha_n f_n \tag{37}$$

$$\lambda \frac{df_n}{d\xi} = 0 \quad \text{at} \quad \xi = 0 \quad \text{and} \quad \xi = 1 \tag{38}$$

$\alpha_n$ being the corresponding eigenvalue.

The last term in the right-hand side of eq.(36) plays an important role in the determination of the velocity field $\mathbf{v}$ but its effect is limited to relatively short periods of weak currents (at tide reversal, for instance) (Nihoul, 1977; Nihoul *et al.*, 1979) and it contributes very little to the residual turbulent Reynolds stress obtained by averaging over a time T covering several tidal periods.

Hence, setting $z = x_3 + h$, one may write, with a good approximation

$$\tau_0 \sim \tau_0^b + \left(\frac{\tau^s - \tau^b}{H}\right)_0 z \tag{39}$$

(ii) the bottom stress $\tau^b$ is a function of $\tau^s$, the depth-averaged velocity $\bar{u}$ and the time derivaties of $\bar{u}$.

If one excepts, again, short periods of weak currents, $\tau^b$ can be approximated by the classical "quadratic bottom friction law"

$$\tau^b = D \, \|\bar{u}\| \, \bar{u} \tag{40}$$

where $D$ is the drag coefficient.

Averaging over a time $T$ as before, one obtains then

$$\tau_0^b \sim D \, \|\bar{u}_1\|_0 \; \bar{u}_0 + (D \, \|\bar{u}_1\| \; \bar{u}_1)_0 \tag{41}$$

*i.e.*, using eqs. (17) and (19)

$$\tau_0^b \sim H_0 K \, \bar{u}_0 + \tau_0^f \tag{42}$$

Changing variable to $z$ and using (39), one can write

$$\begin{aligned} \frac{1}{H_0} \int_{-h}^{\zeta_0} \tau_0 \cdot \frac{\partial \mathbf{v}_0}{\partial x_3} \, dx_3 &\sim \frac{1}{H_0} \int_{z_0}^{H_0} \tau_0 \cdot \frac{\partial \mathbf{u}_0}{\partial z} \, dz \\ &\sim \frac{1}{H_0} \int_{z_0}^{H_0} \tau_0^b \cdot \frac{\partial \mathbf{u}_0}{\partial z} \, dz + \frac{1}{H_0} \int_{z_0}^{H_0} \left(\frac{\tau^s - \tau^b}{H}\right)_0 \cdot \frac{\partial \mathbf{u}_0}{\partial z} \, z \, dz \end{aligned} \tag{43}$$

The horizontal velocity $\mathbf{u}$, however it may for the rest vary with depth, always has a logarithmic profile near the bottom. This implies that $\frac{\partial \mathbf{u}}{\partial x_3}$ behaves like $z^{-1}$ near $z = 0$. A first consequence of this asymptotic behaviour of the velocity profile is that integrals over depth are, strictly speaking, not taken from $z = 0$ to the surface but from some very small height $z_0$ (the "rugosity length") to the surface. This has been taken into account in eq. (43). A second consequence is that the first integral in the right-hand side of eq. (43) is largely dominant, the singularity at $z = 0$ being cancelled in the second integral by the factor $z$.

Since the second integral is only a small correction, one may make the approximation

$$\left(\frac{\tau^s - \tau^b}{H}\right)_0 \sim \frac{\tau_0^s - \tau_0^b}{H_0}$$

Eq. (43) can thus be rewritten

$$\frac{1}{H_0}\int_{-h}^{\zeta_0} \tau_0 \cdot \frac{\partial \mathbf{v}_0}{\partial x_3}\, dx_3 \sim \frac{\tau_0^b}{H_0}\cdot\int_{z_0}^{H_0} (1 - \frac{z}{H_0})\, \frac{\partial \mathbf{u}_0}{\partial z}\, dz + \frac{\tau_0^s}{H_0}\cdot\int_{z_0}^{H_0} \frac{z}{H_0}\, \frac{\partial \mathbf{u}_0}{\partial z}\, dz \tag{44}$$

Integrating by parts, one gets

$$\frac{1}{H_0}\int_{-h}^{\zeta_0} \tau_0 \cdot \frac{\partial \mathbf{v}_0}{\partial x_3}\, dx_3 \sim \frac{\tau_0^b . \overline{\mathbf{u}}_0}{H_0} + \frac{\tau_0^s . (\mathbf{u}_0^s - \overline{\mathbf{u}}_0)}{H_0} \tag{45}$$

*i.e.*, using eq. (42)

$$\frac{1}{H_0}\int_{-h}^{\zeta_0} \tau_0 \cdot \frac{\partial \mathbf{v}_0}{\partial x_3}\, dx_3 \sim K\, \overline{\mathbf{u}}_0^2 + \frac{\tau_0' . \overline{\mathbf{u}}_0}{H_0} + \frac{\tau_0^s . (\mathbf{u}_0^s - \overline{\mathbf{u}}_0)}{H_0} \tag{46}$$

The first two terms in the right-hand side of eq. (30) integrated over depth give then

$$\int_{-h}^{\zeta_0} [\frac{\partial}{\partial x_3}(\mathbf{v}_0 . \tau_0) - \tau_0 \cdot \frac{\partial \mathbf{v}_0}{\partial x_3}]\, dx_3 = \int_{-h}^{\zeta_0} \mathbf{v}_0 \cdot \frac{\partial \tau_0}{\partial x_3}\, dx_3 \sim \tau_0^s . \overline{\mathbf{u}}_0 - K H_0 \overline{\mathbf{u}}_0^2 + \tau_0' . \overline{\mathbf{u}}_0 \tag{47}$$

*i.e.* the same result one would have obtained from the depth integrated transport equation for the contribution of the wind stress and the bottom stress, by taking the scalar product of eq. (16) by $\overline{\mathbf{u}}_0$.

One notes that, in the right-hand side of eq. (46), only the first term can be associated without ambiguity to the dissipation of energy. The second term represents the rate of work of the mesoscale friction stress. Although its "bottom friction" origin is clear, its sign cannot be set *a priori* and there is no reason why it could not actually provide energy to the residual flow.

The same can be said for the last term in the right-hand side of eq. (30). This term appears with the opposite sign in eq. (31). It thus represents an exchange of energy between residual and mesoscale flows; this term can be either positive or negative. There is no way of knowing *a priori* whether the energy is extracted from the mean flow and goes from macroscales to mesoscales or if it is supplied to the mean flow by mesoscale motions.

## The exchange of energy between scales of motion

The exchange of energy between macroscales and mesoscales can be characterized, at each point of the North Sea, by the depth-averaged rates of energy transfer

$$\varepsilon_N = \frac{1}{H_0} \int_{-h}^{\zeta_0} (\mathbf{N} : \nabla \mathbf{v}_0)\, dx_3 \tag{48}$$

$$\varepsilon_F = \frac{1}{H_0} (\tau_0^f . \overline{u}_0) \tag{49}$$

These quantities, which may be positive or negative should be compared with the rate of energy dissipation by the residual motion : i.e.

$$\varepsilon_D = K\, \overline{u}_0^2 \tag{50}$$

The mesoscale Reynolds stress tensor $\mathbf{N}$ also contributes to the term $\nabla.(-\mathbf{N}.\mathbf{v}_0)$ in the left-hand side of eq. (30).

This is a completely different effect because it implies a flux of energy in physical space while $\mathbf{N} : \nabla \mathbf{v}_0$ , appearing in boths eq. (30) and (31), represents a transfer of energy between scales, *i.e.* a flux of energy in Fourier space.

This effect cannot however be ignored if one wants to understand the mechanisms by which the mesoscale stresses act on the residual flow. One shall define

$$\delta = \frac{1}{H_0} \int_{-h}^{\zeta_0} [\nabla.(-\mathbf{N}.\mathbf{v}_0)]\, dx_3 \tag{51}$$

It may be noted here that

$$H_0(\delta + \varepsilon_N) = \int_{-h}^{\zeta_0} [\mathbf{v}_0 . (\nabla.\mathbf{N})]\, dx_3$$

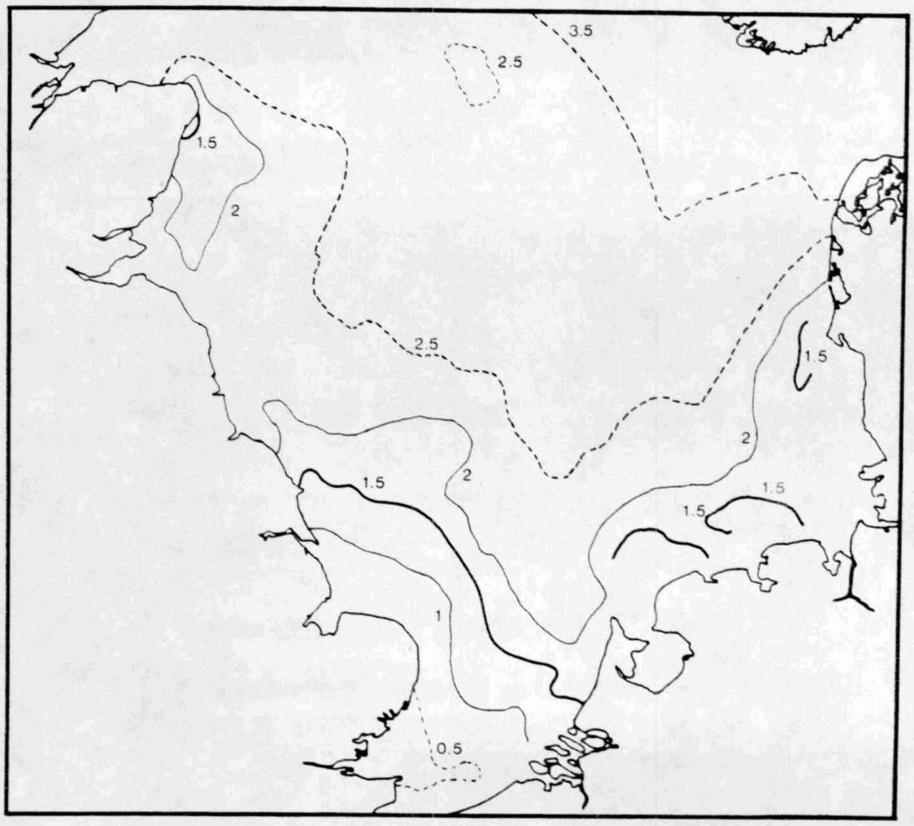


fig. 6.

Curves of equal values of the Simpson-Hunter parameter $S = \log_{10}(10^{-4}\epsilon^{-1})$ in the North Sea.

is not quite the same as the rate of work of the mesoscale Reynolds stress $\boldsymbol{\tau}_0^n$ as one would evaluate it by taking the scalar product of eq.(16) by $\overline{\mathbf{u}}_0$.

It has seemed interesting to explore the interactions between the residual flow and the mesoscale motions in the North Sea by calculating $\varepsilon_N$ , $\varepsilon_F$ , $\varepsilon_D$ and $\delta$ using the results of the three-dimensional model mentioned earlier (Nihoul, 1977; Nihoul *et al.*, 1979). Taking, to begin with, a situation of negligible wind forcing, the model was applied to the determinations of the tidal flow, of the tidal residuals and of the transfer functions $\varepsilon_N, \varepsilon_F, \varepsilon_D$ and $\delta$. The depth-averaged dissipation rate $\varepsilon$ (eq. 34) was also computed for comparison.

Figure 6 shows the distribution over the North Sea of the non-dimensional parameter

$$S = \log_{10} \frac{\varepsilon^*}{\varepsilon} \tag{52}$$

where $\varepsilon^*$ is a value of reference taken here as $10^{-4}\ m^2.s^{-3}$.

The parameter S was introduced by Simpson and Hunter who argued that the critical value S = 1.5 indicated the regions where fronts were likely to form in the summer. This prediction appears to be fairly well confirmed by observations (e.g. Pingree and Griffiths 1978, Nihoul 1980).

One can see on figure 6 that $\varepsilon$ varies from values of the order of $10^{-5}$ or larger in the Southern Bight to $10^{-6}$ in coastal areas and $10^{-7}$ in the Northern part of the North Sea.

There is some similitude between the distribution of $\varepsilon$ and that of $\varepsilon_D$, $\varepsilon_N$ and $\delta$ but $\varepsilon_D$, $\varepsilon_N$ and $\delta$ are in general about two orders of magnitude smaller than $\varepsilon$. Furthermore, while $\varepsilon_D$ is positive definite, both positive and negative values of $\varepsilon_N$ and $\delta$ are found.

Positive values of $\varepsilon_N$ indicate a transfer of energy from the mean (residual) flow to the mesoscale motion (a positive mesoscale eddy viscosity in the terminology of turbulence). Such positive values are found in rather well-marked often isolated regions which have the appearance of large mesoscale eddies with smaller mesoscale eddies inside them. This can be seen on figure 7 which shows the distribution of the positive values of

$$\hat{\varepsilon}_N = \frac{\varepsilon_N}{\varepsilon_D} \tag{53}$$

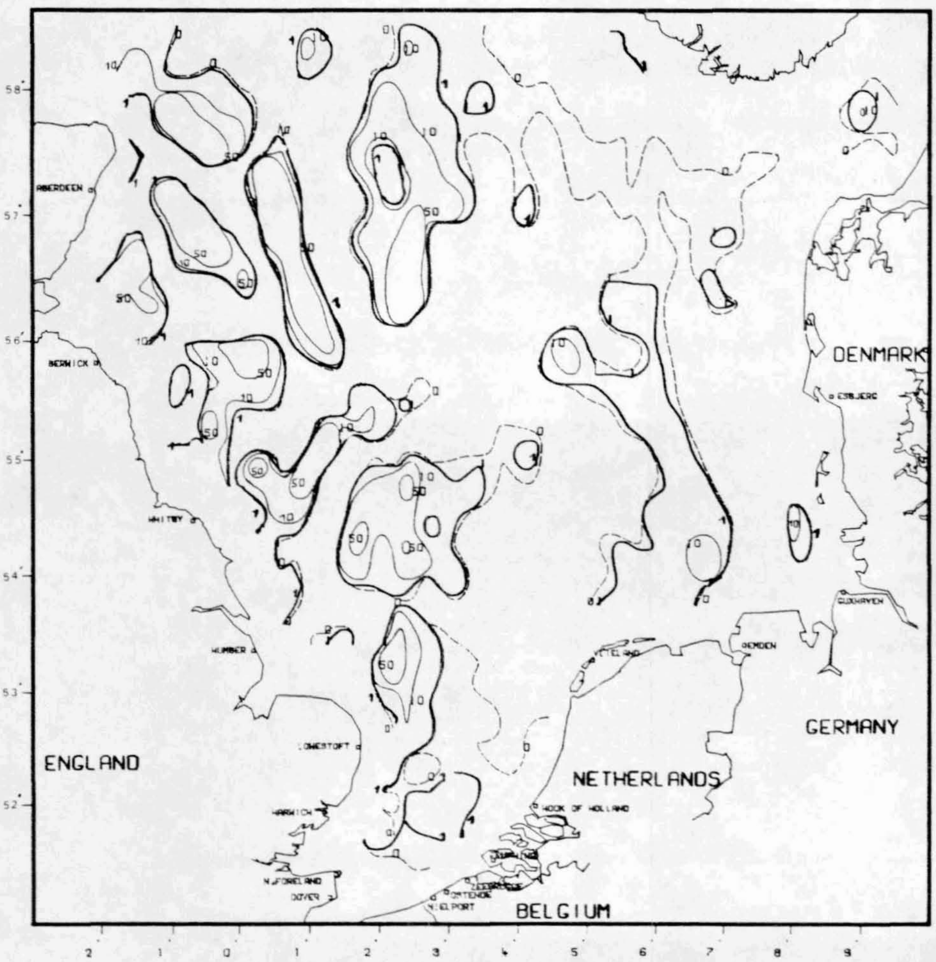


fig. 7.

Distribution of the positive values of the normalized transfer function $\hat{\epsilon}_N$ in the North Sea (no wind).

The heavy line is the curve $\hat{\epsilon}_N = 1$. In many places, it nearly coincides with the line "0" where $\epsilon_N$ changes sign*.

* In interpreting figure 7 and those which follow, one must remember that having to calculate horizontal gradients, the model can only provide results one grid point away from the coast. One cannot say anything from the figures about the coastal fringe.

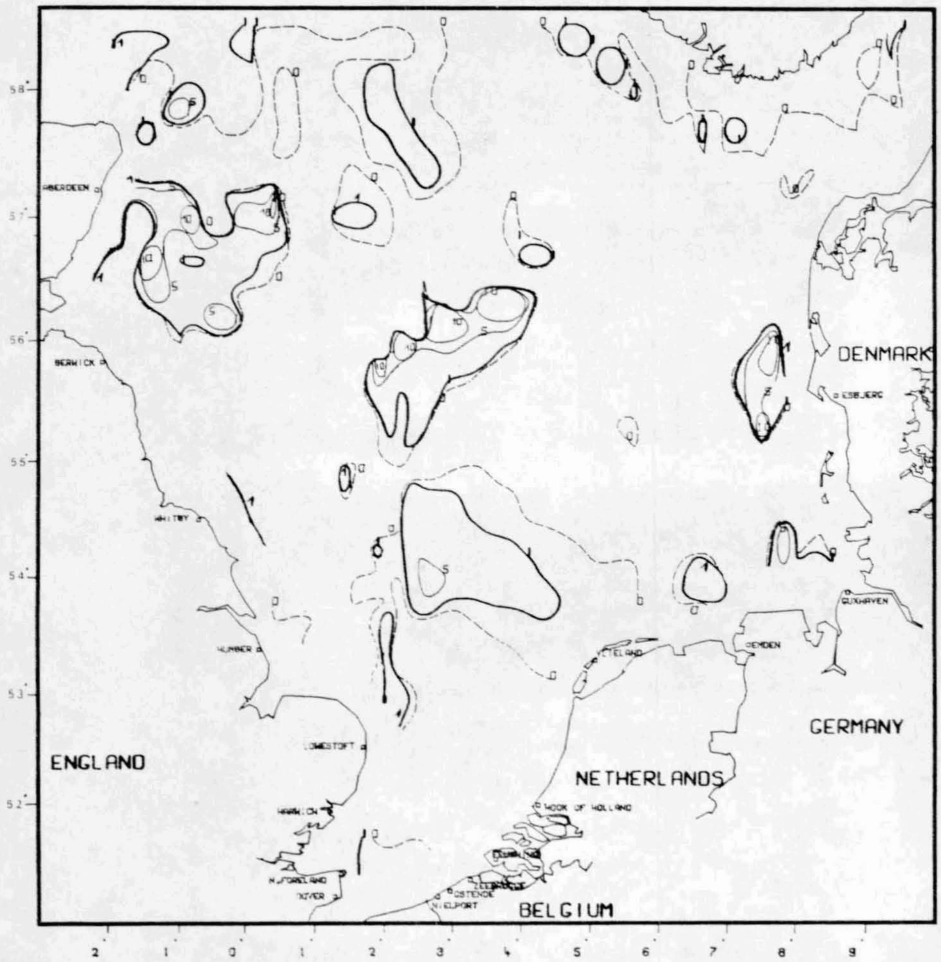


fig. 8.

Distribution of the positive values of the normalized function $\epsilon_N + \delta$ in the North Sea (no wind).

The distribution of $\delta$ is roughly the opposite. $\delta$ is generally negative where $\varepsilon_N$ is positive. As a result the sum $\varepsilon_N + \delta$ drops by almost one order of magnitude as shown in figure 8 where the positive values of

$$\hat{\varepsilon}_N + \hat{\delta} = \frac{\varepsilon_N}{\varepsilon_D} + \frac{\delta}{\varepsilon_D} \tag{54}$$

are plotted (the heavy line is the curve $\hat{\varepsilon}_N + \hat{\delta} = 1$). Again the positive regions appear rather localized while the negative values are more diffused, the largest negative values (in the range $-1, -10$) occuring in the western and central parts of the North Sea.

Positive and negative values of $\varepsilon_F$ are also found in different regions of the North Sea. However these values are generally small $(\sim 10^{-9})$ except in a few localized places such as very shallow areas like the Southern Bight, where positive and negative values of $\varepsilon_F$ of the order of $10^{-7}$ are observed. (This result could have been anticipated. In the absence of wind, the mesoscale velocity $\overline{u}_1$ is the tidal velocity and the contribution to $\tau_0^f$ from a velocity $\overline{u}_1$ at a given time tends to be cancelled by an opposite contribution of a velocity $-\overline{u}_1$, about one half tidal period later. In shallow waters of course, the smallness of the depth, appearing at the denominator in eq. (49), increases the order of magnitude of $\varepsilon_F$.)

The sum $\varepsilon_N + \varepsilon_F$ represents the rate of work of the mesoscales stresses. Again, one finds regions where it is positive and regions where it is negative with rather sharp transitions indicated as before by the frequent coincidence of the curves 0 and 1 in the plot of the normalized rate of work

$$\hat{\varepsilon}_N + \hat{\varepsilon}_F = \frac{\varepsilon_N + \varepsilon_F}{\varepsilon_D} \tag{55}$$

Positive and negative values of $\hat{\varepsilon}_N + \hat{\varepsilon}_F$ are found in the range 0 - 10 with large regions where it is of order 1.

The pattern of the total rate of energy production (or destruction).

$$\hat{\varepsilon}_N + \hat{\varepsilon}_F + 1 = \frac{\varepsilon_N + \varepsilon_F + \varepsilon_D}{\varepsilon_D} \tag{56}$$

is rather similar to the pattern of $\hat{\varepsilon}_N + \hat{\varepsilon}_F$. (Because of the sharp transitions, the addition of 1 is determinant only in a few places.)

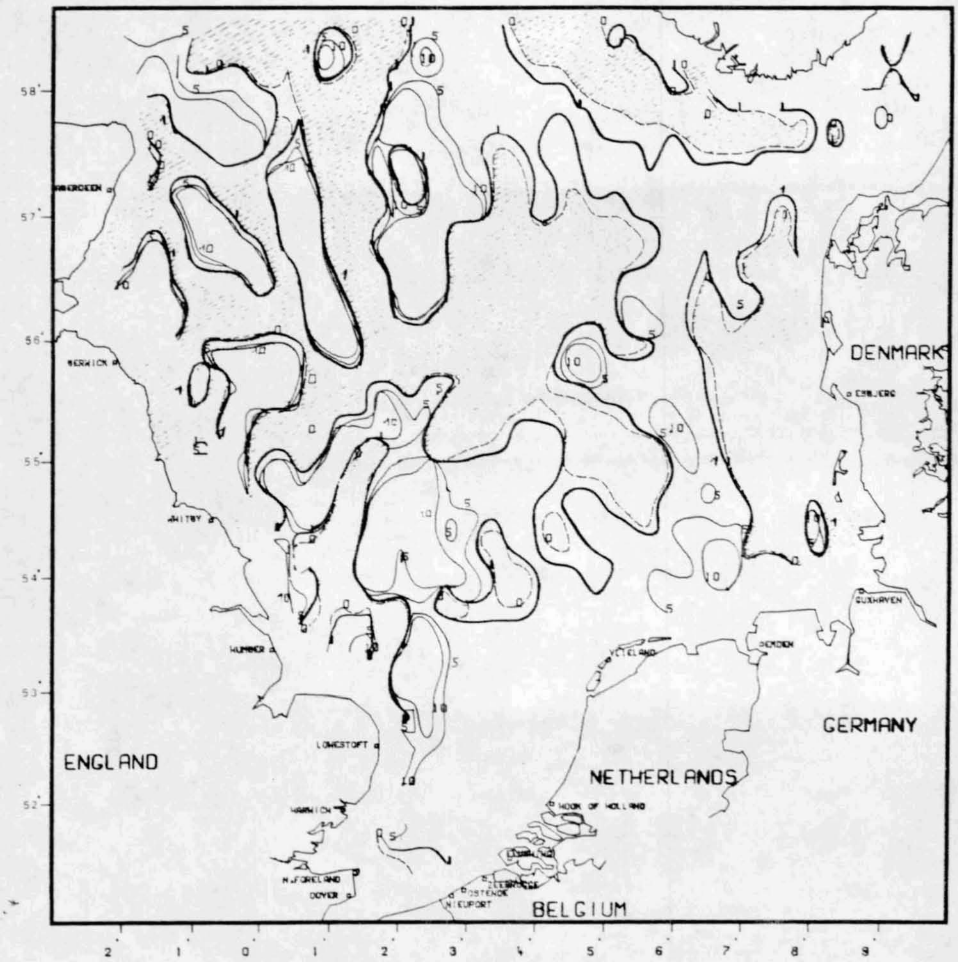


fig. 9.
Distribution of positive values of $1 + \hat{\epsilon}_N + \hat{\epsilon}_F$ in the North Sea (no wind)

The two patterns are less patchy. Regions of positive and negative values form more connected zones.

Comparing with the corresponding residual circulation, one finds that well-identified residual gyres are generally associated with regions of negative values (negative mesoscale eddy viscosity in turbulent terminology) while streamwise flows appear rather to follow the bands of positive values (positive mesoscale eddy viscosity in turbulent terminology) in agreement with the results of an earlier study (Nihoul, 1980).(Figure 9).

In summary, the interaction between the residual flow and the mesoscale tidal flow appears to be characterized by a transfer of energy between motions of different scales. This transfer goes from macroscales to mesoscales in some regions (positive mesoscale eddy viscosity effect) and from mesoscales to macroscales in some other regions (negative mesoscale eddy viscosity effect). A horizontal flux of kinetic energy is set up to compensate, to some extent.

It is interesting to note that the total sum

$$\hat{\delta} + \hat{\varepsilon}_N + \hat{\varepsilon}_F + 1 = \frac{\delta + \varepsilon_N + \varepsilon_F + \varepsilon_D}{\varepsilon_D} \qquad (57)$$

is not positive everywhere. A rather extensive patch of negative values (in the range -1 to -10)spreads out from the Western North Sea, off the coasts of Scotland and Northern England, into the central part of the North Sea.

Going back to eq. (30) where the term $\nabla.(\mathbf{v}_0 \frac{v_0^2}{2})$ is always completely negligible, one sees that, in the absence of wind forcing, negative values of the sum (57) imply

$$\nabla.(\mathbf{v}_0 q_0) = \mathbf{v}_0.\nabla q_0 > 0 \qquad (58)$$

*i.e.* the mesoscale stresses are actually driving the residual flow "up the residual slope.and pressure gradient".

## The effect of the wind

As pointed out, in the introduction, the tidal residuals considered in the preceding sections can only constitute a first approximation of what a real climatic residual circulation is. The atmospheric forcing has been neglected both in determining

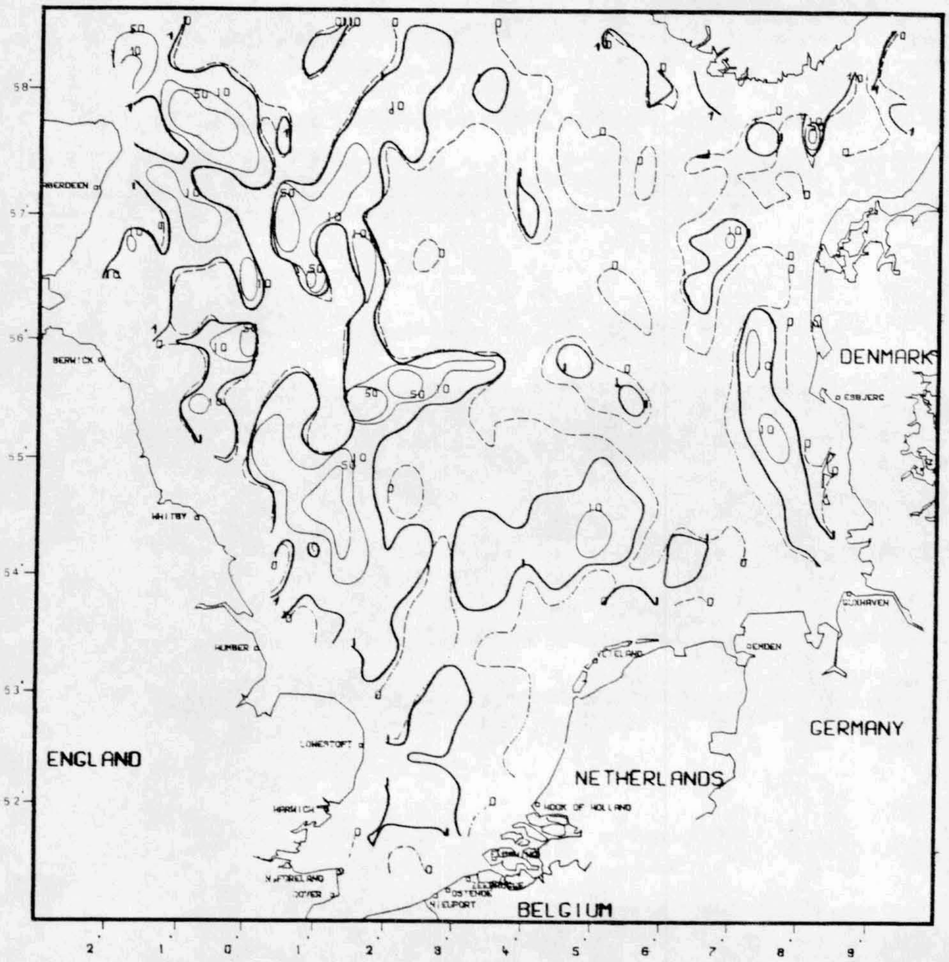


fig. 10.

Distribution of positive values of $\tilde{\epsilon}_N$ in the North Sea (uniform constant wind of 15 $m.s^{-1}$ from the North-West)

the mesoscale motion and in computing the resulting residuals. The advantage was that the time of averaging could be limited to two or three tidal periods. This is not possible if one includes the effect of a wind field which itself evolves with a characteristic time of the same order.

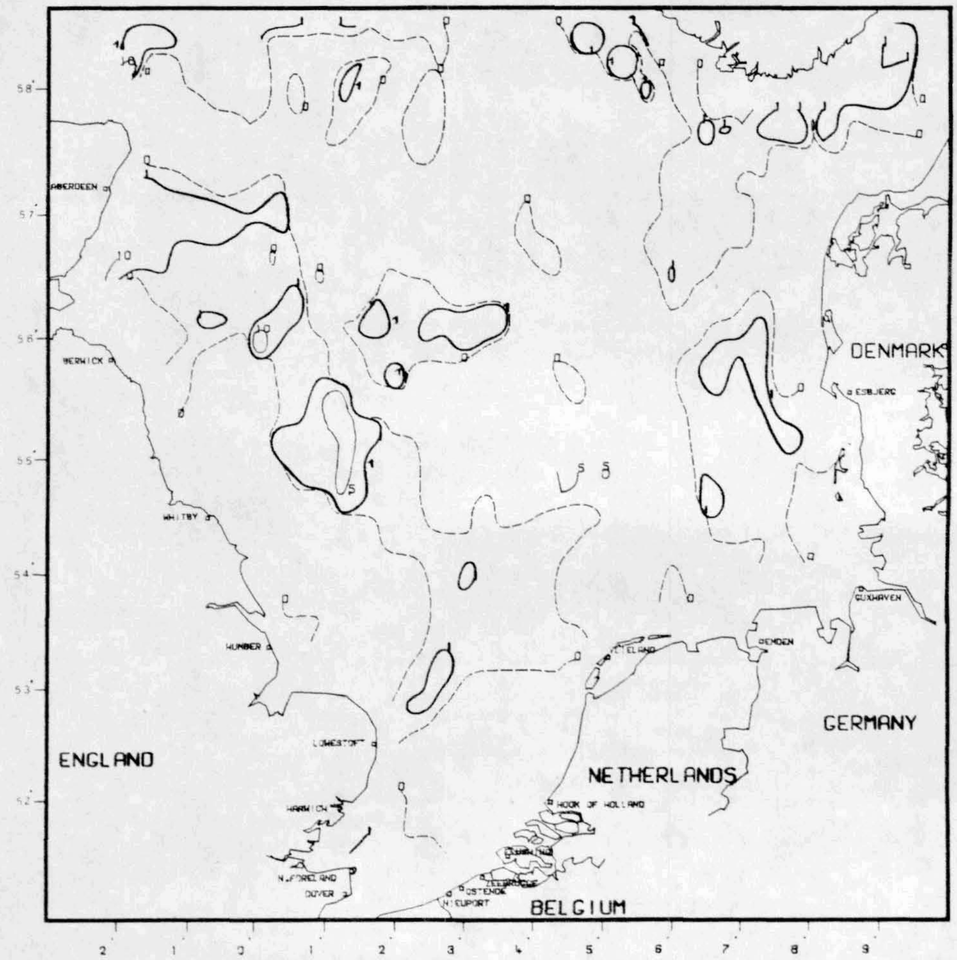


fig. 11.

Distribution of positive values of $\dot{\varepsilon}_N + \dot{\delta}$ in the North Sea
(uniform constant wind of 15 m.s$^{-1}$ from the North-West)

In some cases, one may have to go to averaging over several weeks to ensure that the average is meaningful. This implies that the mesoscale velocity field must be calculated over the same period of time and the cost of operating the model becomes rapidly prohibitively large.